Badminton Projectile Motion: Modeling Air Resistance

Introduction: This activity requires Tracker (download it if you do not already have a version on your computer) and the Coriolis.trk and Coriolis.mov files on ComPADRE: http://www.compadre.org/osp/items/detail.cfm?ID=12086. It is based on the following videos:

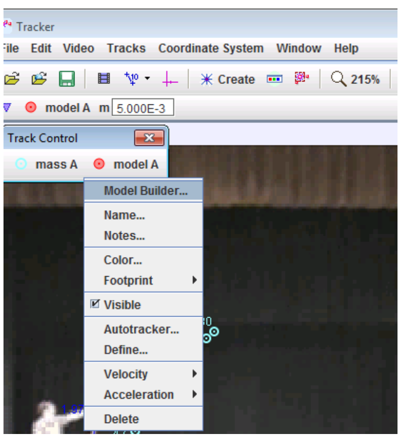
- From the Doane College Physics Department Video Library: http://physics.doane.edu/physicsvideolibrary/default.html#projectile, Faculty: Mark Plano Clark and Chris Wentworth.
- YouTube video at http://www.youtube.com/watch?v=NzKdQKoNbLc.

You will use both movies and two pre-marked Tracker files.

Part A: Open shuttlecock_model.trk in Tracker. Play the video. Notice that there are two "Tracks":

1. Mass A which follows the birdie and was created by clicking on the position of the birdie at each frame of the video.
2. Model A which is a dynamical (mathematical) model of the motion.

Click on "model A" and choose "Model Builder." What are the force equations that the model uses?



F_x = ________________________

F_y = ________________________

Is this the correct equation for projectile motion?

What are the initial conditions for the model?

Do the initial conditions match the initial position and velocity of the birdie (in the plot view, you can switch from plots of "mass A" and "model A" as well as clicking on the axis labels of the plots to change what is plotted)?

Most students find that the model is properly built for projectile motion with the correct initial conditions, so why doesn't the model match the motion? What have we neglected (that is pretty important in badminton)?

Air resistance is the missing part of the model. There are two common models of air resistance one where the air resistance is proportional to velocity and one where it is proportional to velocity squared:

$$|F| = -bv\ (linear)$$

$$|F| = -bv^2\ (quadratic)$$

The research on badminton birdie's shows that the quadratic air resistance best explains birdies falling straight down and in projectile motion,[1] so we will build a model using quadratic air resistance and we need to find the vector components:

$\vec{F} = -bv^2\hat{v}$ where $\hat{v}$ means the unit vector in the direction of the velocity. Explain why it can be written as follows:

$$\hat{v} = \frac{v_x\hat{x} + v_y\hat{y}}{\sqrt{v_x^2 + v_y^2}}$$

Explanation (show it is indeed a unit vector, explain why it gives the correct direction).

It may be helpful to remember that $v = \sqrt{v_x^2 + v_y^2}$:

Therefore, explain why the components of the force equation are as follows (in Tracker's format):

F_x=-b*vx*sqrt(vx*vx+vy*vy)

F_x=-m*g-b*vy*sqrt(vx*vx+vy*vy)

Explanation or show that this is equivalent to the vector expression $\vec{F} = -bv^2\hat{v}$. It may be helpful to remember that $v = \sqrt{v_x^2 + v_y^2}$:

Enter the equations in the Model Builder for Model A. Adjust the value of b until the model matches the motion.

b = ______________________

[1] Chen, Pan and Chen, "A study of shuttlecock's trajectory in badminton," *Journal of Sports Science and Medicine* (2009), **8**, 657-662. and Peastrel, Lynch and Angelo, "Terminal velocity of a shuttlecock in vertical fall, *American Journal of Physics* (1980), **48**, 511-513.

Part B (Optional Extension):

When objects fall and air resistance is substantial, there are times when the object falls at a constant speed, the terminal velocity. For an object falling at terminal velocity, what is the acceleration?

In a force diagram, then, an object falling with terminal velocity experiences the downward gravitational force and an upward (against the direction of velocity) air drag and these forces are equal. Thus, find an equation for the drag coefficient, b, as a function of the mass, acceleration due to gravity and the terminal velocity (show your work):

b=

In **shuttlecock_model**, if you look at the data, although you can tell that the velocity is not changing much at the end it is hard to make the claim that the velocity is constant.

Open shuttlecock2_model.trk. Here the birdie is more difficult to follow. Nevertheless, you can get a pretty clear picture of the x and y positions versus time (look at the plots). Clearly air resistance is important. What happens to the x-position at the end of the trajectory?? What is the approximate velocity in the x-direction?

What about the y-position and velocity at the end of the trajectory? What in the plots indicates that the birdie may have reached terminal velocity at the end of the trajectory?

What is the (approximate value) of terminal velocity: _____________

Therefore, what value (approximately) should you use for b (show your work):

b= ____________

Complete the model and check the value. (The most likely sources of error are the unknown scale: used a height of 1.8m for the person hitting the birdie to scale the video and the moving camera which has been adjusted for as much as possible by adjusting the origin in each frame).